5 - Sets 2

Example. Suppose A = {a, b, c, d, e} , B = {d, e, f}, C = {1, 2, 3}. Then:

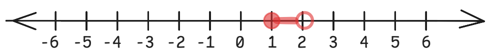
- $A \cup B$ = {a, b, c, d, e, f}
- $A \cap B$ = {d, e}
- $A - B$ = {a, b, c}
- $B - A$ = {f}
- $(A - B) \cup (B - A)$ = {a, b, c, f}
- $A \cap C$ = { } (empty set)
- $A - C$ = {a, b, c, d, e}

Exercise. Suppose A = {4, 3, 6, 7, 1, 9}, B = {5, 6, 8, 4} and C = {5, 8, 4}. Find:

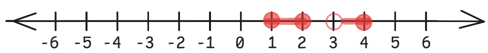
1. $A \cap B$ = {4, 6}
2. $A - B$ = {3, 7, 1, 9}
3. $A - C$

Exercise. Graph the following sets on the number line:

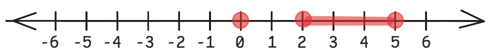
1. $[0, 5] \cap (0, 2) \cap [1, 3]$



2. $(A \cup B) \cap C$ if $A = [0, 2], B = (3, 5), C = [1, 4]$



3. $[0, 5] - (0, 2)$



Notation. We write $\{x : P(x)\}$ instead of $\{x \in U : P(x)\}$ if the universe U is clear from context.

So the set of 2-digit square numbers can also be written as:

$$\{x^2 : x \in \mathbb{N} \text{ and } 10 \leq x \leq 99\}$$

Cardinality

Definition. The **cardinality** or **size** of a set S, denoted |S|, is the number of elements in the set.

Example. Remember that elements are listed within the outer pair of braces, and separated by commas. The cardinality of a set counts.

With curly braces: S = {a, b, 1, 2, Abe}

Box model: [a b 1 2 Abe]

|S| = 5

Sets can also be nested, so the box model of sets help us only extract the elements of a set.

T = { {1, 2}, {2, 4}, a, b} = [[1, 2] [2, 4] a b] => |T| = 4

U = { {2}, {2, 2}, a, b} = {{2}, {2}, a, b} = {{2}, a, b} => |U| = 3

V = {} => |V| = 0

- Is 1 an element of T? No
- Is {1,2} an element of T? Yes
- Is {1,2} a subset of T? No
- Is 2 an element of U? No
- Is {2} a subset of U? No
- Is {2} an element of U? Yes

Example. Find the cardinality of the following sets.

- $\mathbb{Z} \cap [-10, 10]$ = {-10, -9, ..., -1, 0, 1, 2, 3, ..., 8, 9, 10} => size = 21
- $\{5x : x \in \mathbb{N}\} \cap [0, 100]$ = {0, 5, 10, 15, ..., 95, 100} => size = 21, for instance by solving 0 ≤ 5x ≤ 100 for natural number x's.
- $\{x \in \mathbb{N} : x \equiv 0 \pmod{3}\} \cap \{x \in \mathbb{N} : x \equiv 0 \pmod{5}\} \cap [0, 100]$

  1st set: natural numbers divisible by 3.
  2nd set: natural numbers divisible by 5.
  1st intersect 2nd set: natural numbers divisible by 15.
  So this triple intersection = {0, 15, 30, 45, 60, 75, 90} => size = 7

- $\{x \in \mathbb{Z} : x \equiv 1 \pmod{7}\} \cap [0, 100]$

  x leaves remainder 1 when divided by 7 means: x = 7q + 1 (q = quotient).
  So we solve: 0 ≤ 7q+1 ≤ 100 by subtracting 1 and dividing by 7 to get:
  -1/7 ≤ q ≤ 14+(1/7) so q can be 0, 1, 2, ..., 14 => size = 15.

- $\{x \in \mathbb{Z} : x \equiv 3 \pmod{19}\} \cap [-50, 100]$